EE565:Mobile Robotics
Lecture 2

Welcome

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Organization Lab Course

- **Lab grading policy (40%)**
  - **Attendance = 10 %**
  - **In-Lab tasks = 30 %**
  - **Lab assignment + viva = 60 %**
- **Make a group**
- **Either use lab computers or bring your own laptop (Recommended)**

Today’s Objectives

- Wheel Kinematics and Robot Pose
  - Differential wheel drive
  - Ackermann wheel drive
- Introduction to Mobile Robot Sensors
  - Wheel Encoders
  - Inertial Measurement Unit (IMU) and GPS
  - Range sensors (Ultrasonic, 2D/3D Laser Scanner)
  - Vision sensor (Monocular, Stereo Cameras)
- Introduction to Mobile Robot Actuators
  - DC Brush/Brushless motors
- Motion Controller
  - Position controller
  - PID based Velocity controller

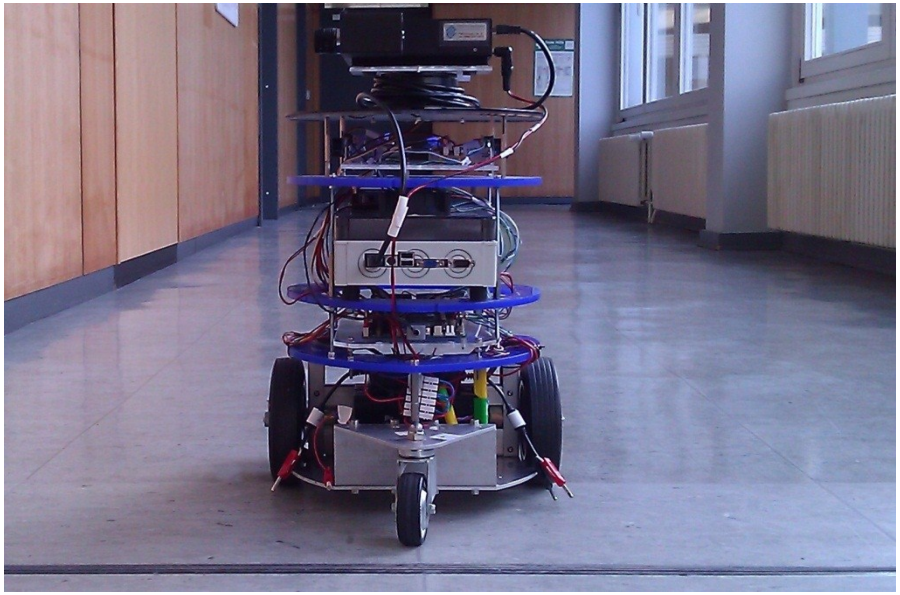
Wheel Kinematics and Robot Pose Calculation

- Basics
- Wheel Kinematics
  - Wheel types and constraints
  - General wheel equation
  - Differential drive robot
- Geometric solutions for wheel kinematics
  - Ackermann steering
  - Double Ackermann steering

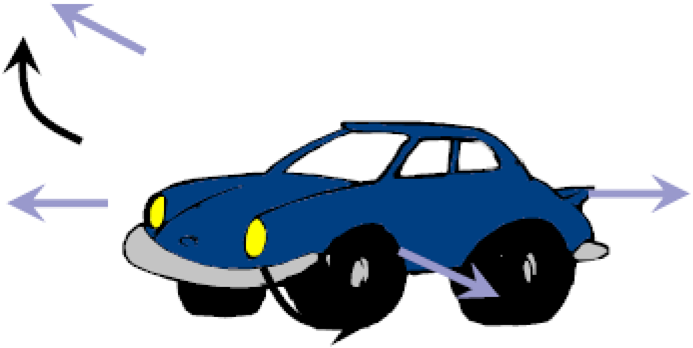
Basics

- Degree of Freedoms (DOF)
- Mobile Robot Pose
- Kinematic
- Kinematics models
- Wheel Types and Constraints
  - Holonomic
  - Non-holonomic

Degree of Freedoms (DOF)



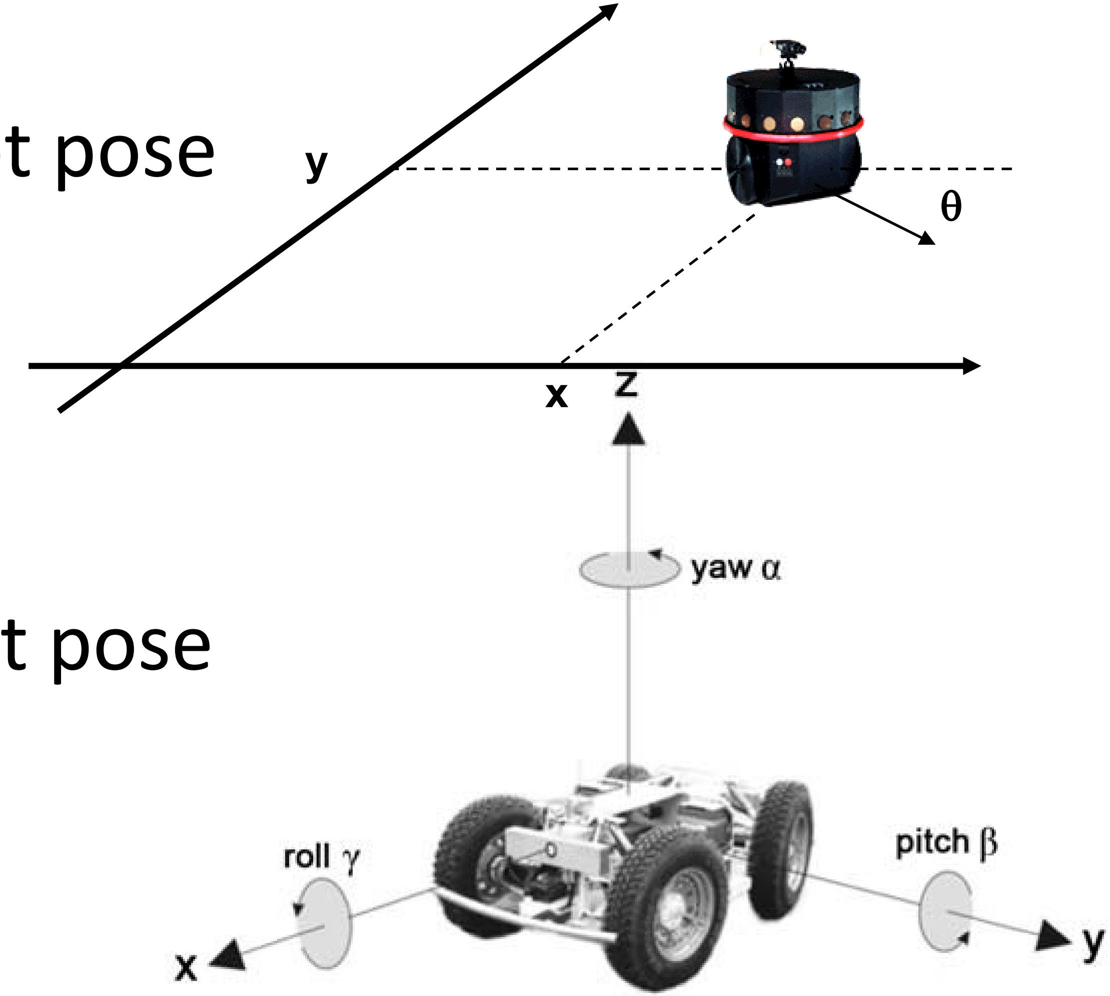
- DOF for a mobile robot are the number of directions in which motion can be made.
- A car has 3 DOF: Translation(2) + Rotation(1)



Pose

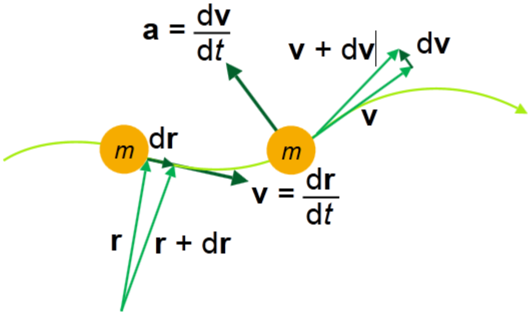
- $X = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$ 2D robot pose

- $X = \begin{bmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{bmatrix}$ 3D robot pose

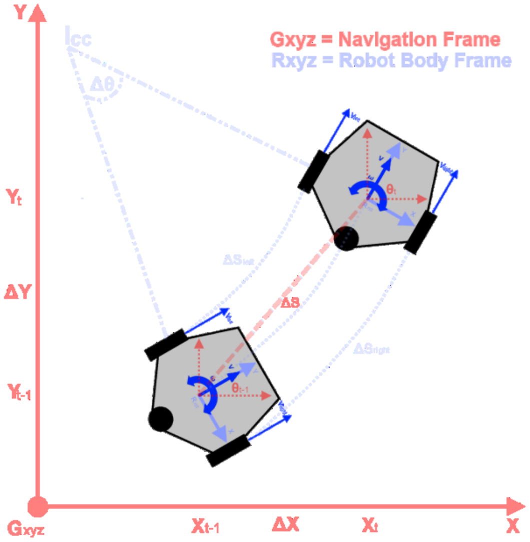


Kinematics

- Kinematics is the study of motion with out the origin of force.

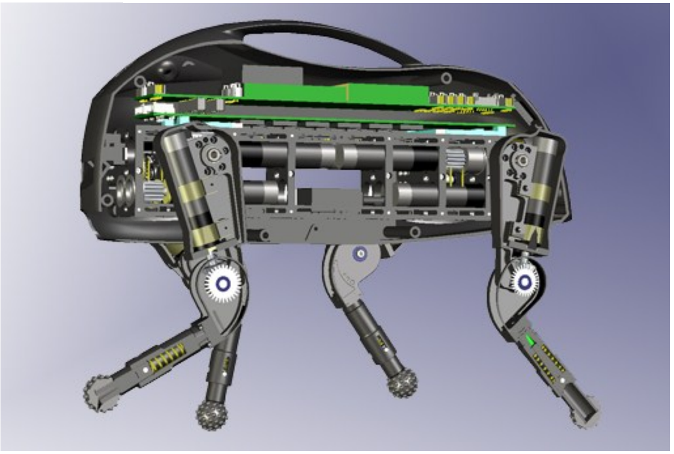


- Mobile robot kinematics deal with the relationship of whee motion and constraints with the platform motion.



Mobile Robot Kinematics

- Legged locomotion
- Wheeled locomotion
  - De facto standard
  - Highly efficient on hard surfaces



Mobile Robot Kinematics (Cont.)

- It is used for position and motion estimation
- A mobile robot moves unbounded in its environment
  - There is no direct way to measure robot’s pose
  - It is integrated over time which leads to inaccuracies
- Each wheel contributes to robot motion and therefore also it’s constraints.

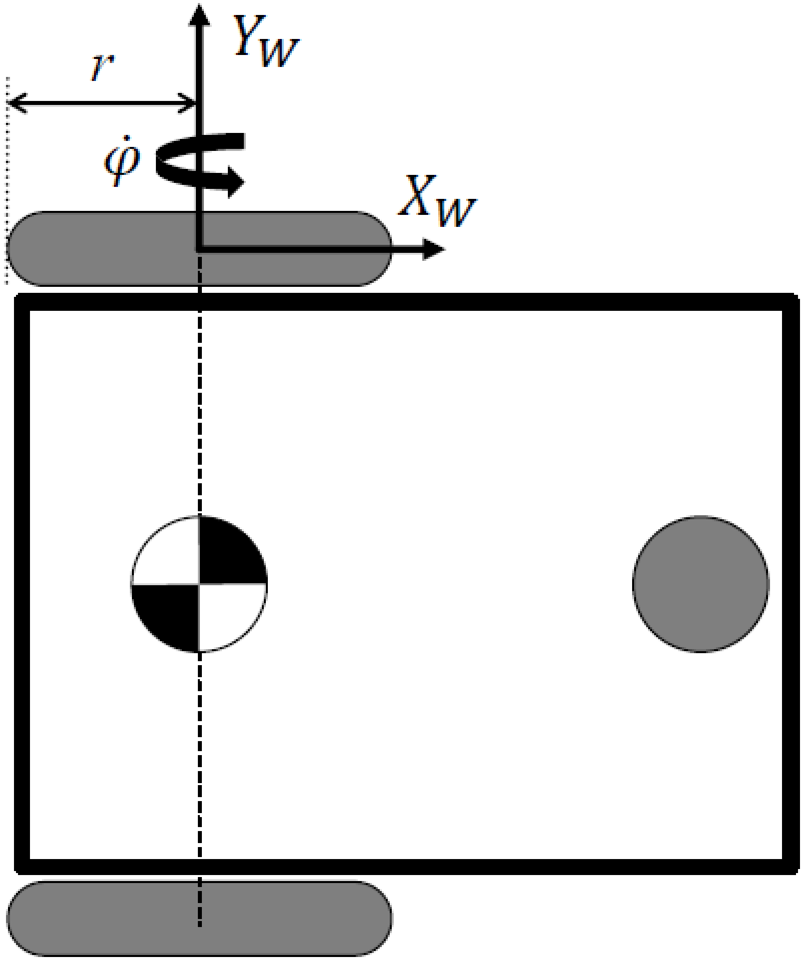
Differential Drive Kinematics

- 3 DOF
- Not all DOF can be actuated or have encoders
- Wheels can impose differential constraints which complicates the computation of Kinematics

Rolling constraint

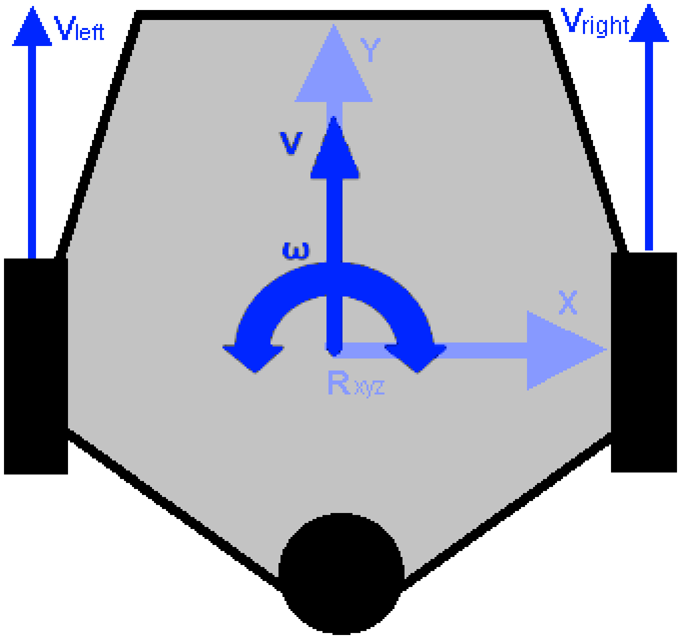
- $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}\dot{\varphi} r\\ 0\end{bmatrix}$

No-sliding constraint



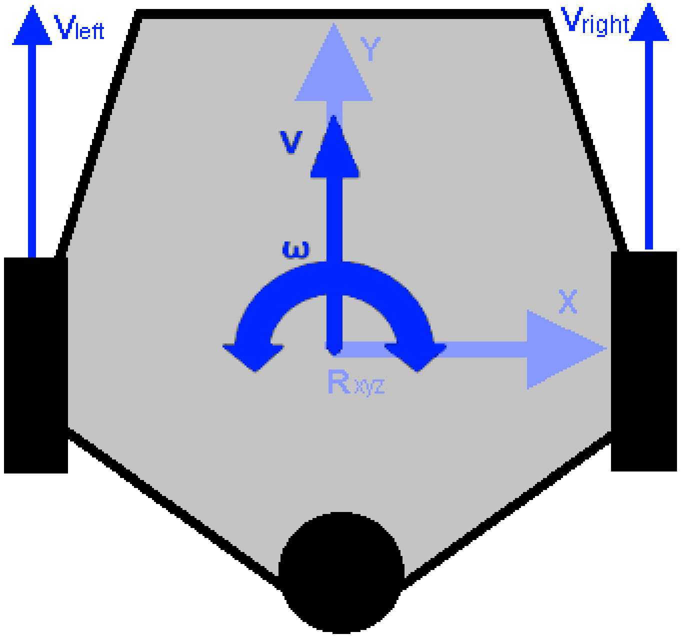
Differential Drive Forward Kinematics

- Forward Kinematics: Given a set of wheel speeds, determine robot velocity



Differential Drive Inverse Kinematics

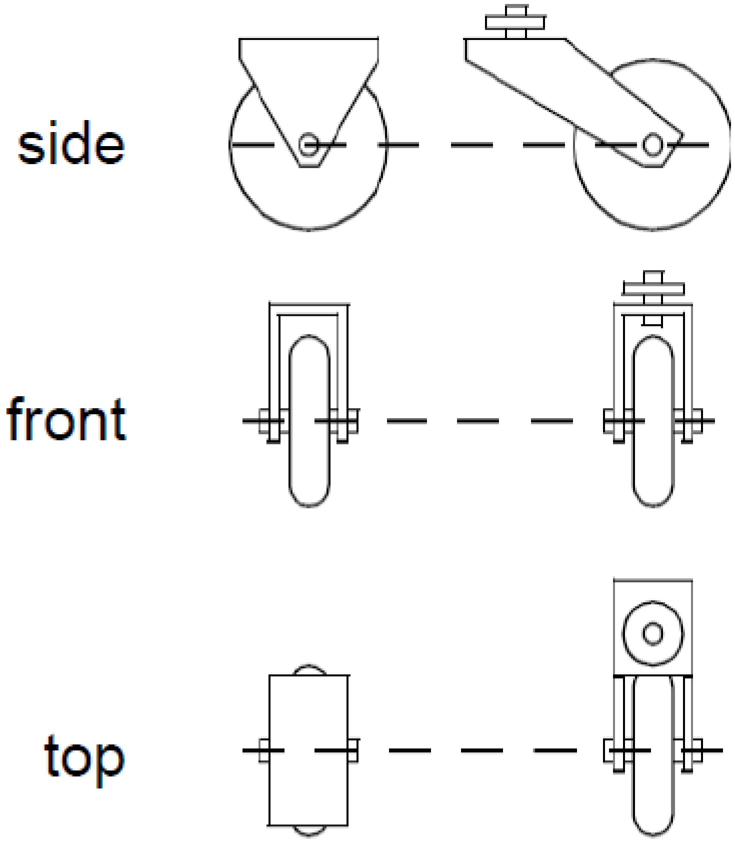
- Inverse Kinematics: Given desired robot velocity, determine corresponding wheel velocities



Holonomic and Non-holonomic

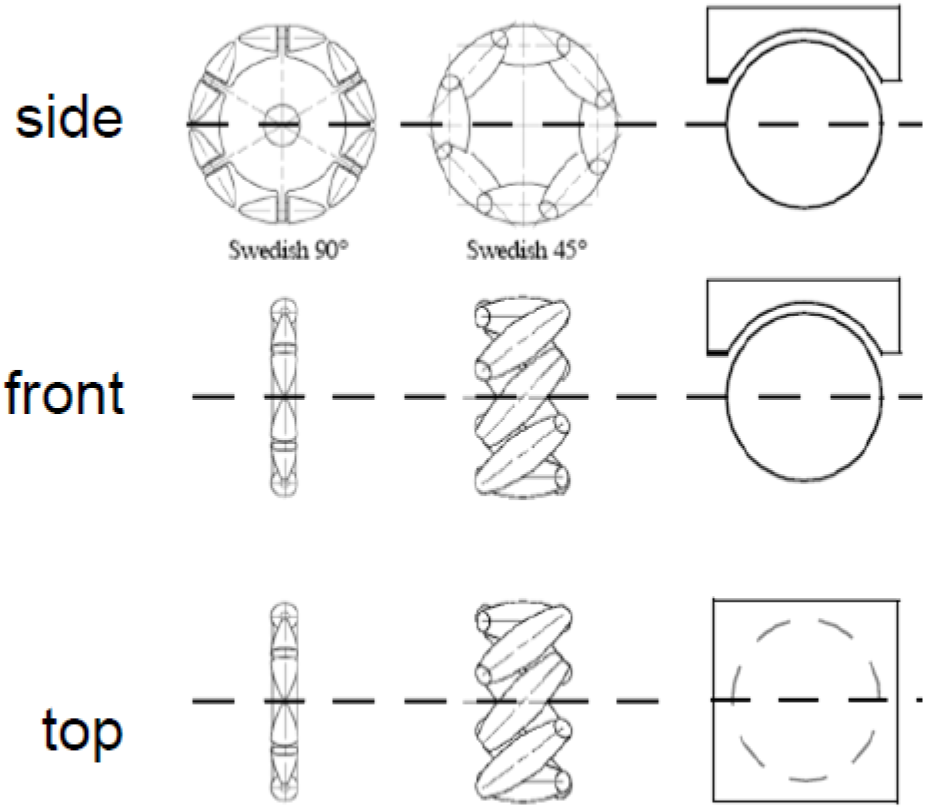
- Holonomic Systems
  - The robot is able to move instantaneously in any direction in the space of its degree of freedom
  - Omnidirectional robot, office chair with castor wheels
- Non-holonomic Systems
  - The robot is not able to move instantaneously in any direction in the space of its degree of freedom
  - Differential drive robot, car

Wheel Types



- Standard Wheels
  - 2 DOF
    - Rotation around wheel axis
    - Rotation around contact point
  - Can be steered/fixed
- Castor Wheels
  - 3DOF
    - Rotation around wheel axis
    - Rotation around contact point
    - Rotation around castor axle

Wheel Types (Cont.)



- Swedish Wheel
  - 3 DOF
    - Rotation around wheel axis
    - Rotation around contact point
    - Rotation around roller
- Spherical Wheels
  - 3 DOF

Wheeled Kinematics

- Problem: For a mobile robot with different wheels, what is the relationship between wheel speed $\dot{\varphi}$ and platform velocities $(\dot{x}, \dot{y}, \dot{\theta})$
- Assumptions
  - Movement on a horizontal plane
  - Point contact of the wheel
  - Pure rolling i.e. no slipping, skidding or sliding
  - Non-deformable wheels
  - No friction for rotation around contact point

Notations

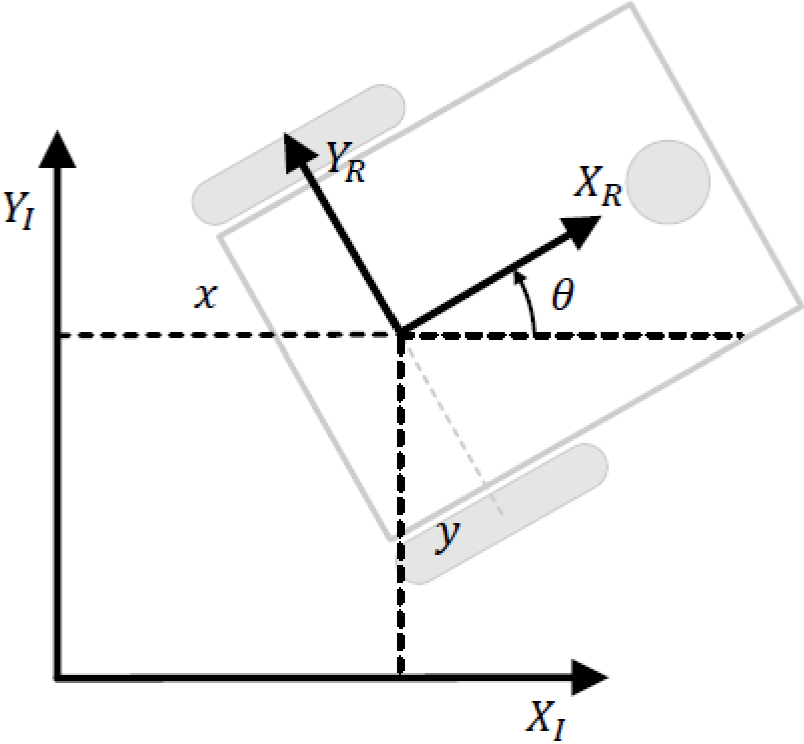
State: $\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$

State velocity: $\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$

$$R(a) = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(a)R(b) = R(a+b)$$

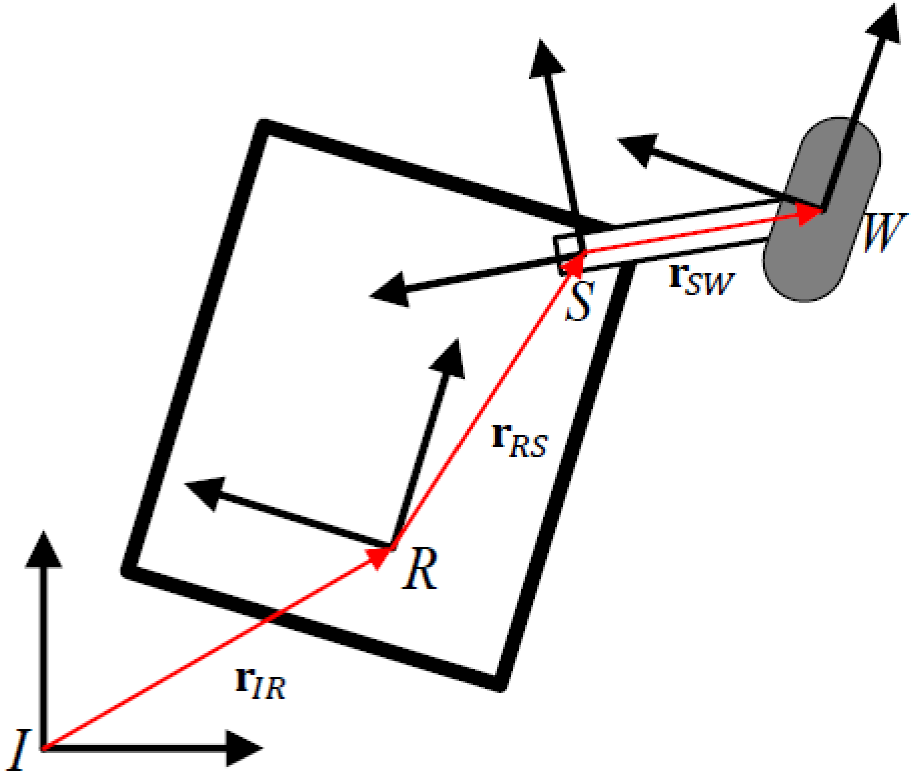
$$\dot{\xi}_R = R(\theta)\dot{\xi}_I$$



Deriving a general wheel equation

- Coordinate Frames
  - I: Inertial
  - R: Robot
  - S: Steering
  - W: Wheel
- The position vector of wheel in inertial frame

$$\mathbf{r}_{IW} = \mathbf{r}_{IR} + \mathbf{r}_{RS} + \mathbf{r}_{SW}$$



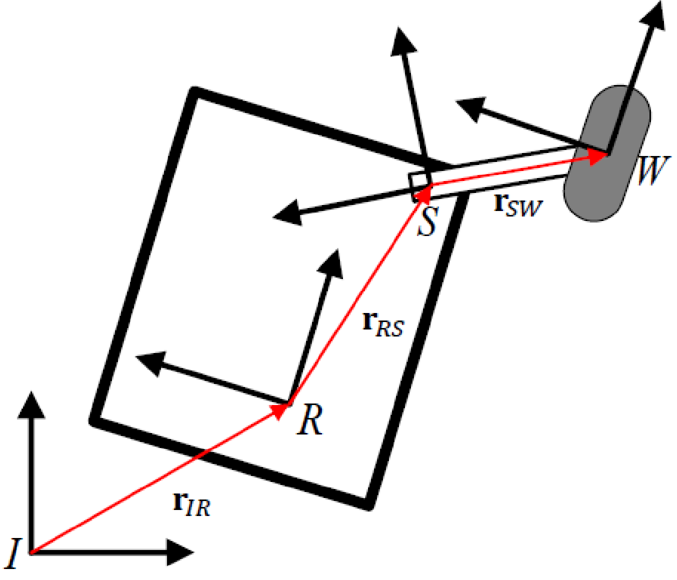
Deriving a general wheel equation (Cont.)

- Velocity vector from one frame to another

$$ {}^{A}\vec{v}_{Q} = {}^{A}_{B}R\,{}^{B}\vec{v}_{Q} \qquad {}^{A}\vec{v}_{Q} = {}^{A}\vec{v}_{OB} + {}^{A}_{B}R\,{}^{B}\vec{v}_{Q} $$

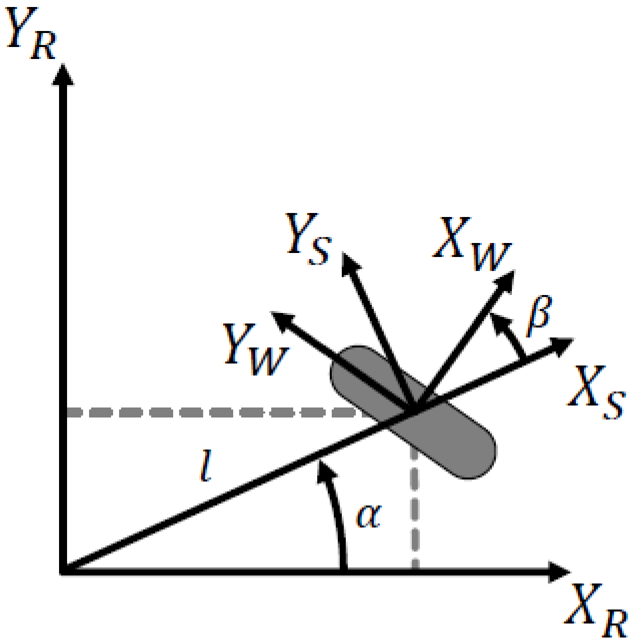
$$ {}^{A}\vec{v}_{Q} = {}^{A}\vec{v}_{OB} + {}^{A}_{B}R\,{}^{B}\vec{v}_{Q} + {}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q $$

$$ \mathbf{v}_{IW} = \underbrace{\mathbf{v}_{IR}}_{\text{robot velocity}} + \overbrace{\boldsymbol{\omega}_{IR} \times \underbrace{\mathbf{r}_{RS}}_{\text{steering offset}} + \boldsymbol{\omega}_{IR}}^{\text{robot angular velocity}} \times \underbrace{\mathbf{r}_{SW} + \overset{\text{steering rate}}{\boldsymbol{\omega}_{RS}} \times \mathbf{r}_{SW}}_{\text{wheel offset}} $$



Standard Wheel (Cont.)

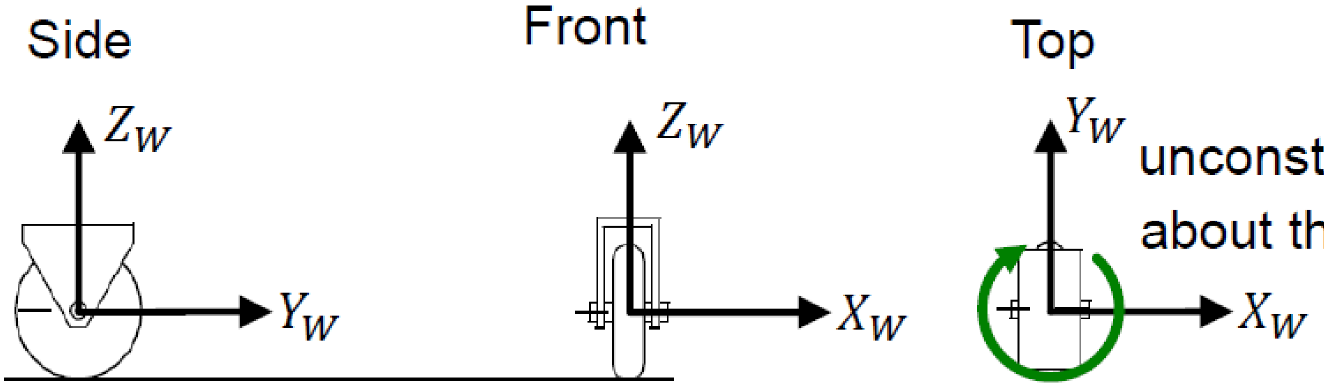
$$\mathbf{v}_{IW} = \underset{\substack{\uparrow\\ \text{robot}\\ \text{velocity}}}{\mathbf{v}_{IR}} + \overset{\substack{\text{robot}\\ \text{angular}\\ \text{velocity}\\ \downarrow}}{\boldsymbol{\omega}_{IR}} \times \underset{\substack{\uparrow\\ \text{steering}\\ \text{offset}}}{\mathbf{r}_{RS}} + \underbrace{\boxed{\overset{\downarrow}{\boldsymbol{\omega}_{IR}} \times \mathbf{r}_{SW}} + \boxed{\overset{\substack{\text{steering}\\ \text{rate}\\ \downarrow}}{\boldsymbol{\omega}_{RS}} \times \mathbf{r}_{SW}}}_{\text{wheel offset} \;=\; 0}$$



$$\mathbf{v}_{IW} = \mathbf{v}_{IR} + \boldsymbol{\omega}_{IR} \times \mathbf{r}_{RS}$$

$\mathbf{v}_{RS} = \dot{\mathbf{r}}_{RS} = \mathbf{0}$ and $\mathbf{v}_{SW} = \dot{\mathbf{r}}_{SW} = \mathbf{0}$

Constraint for a Standard Wheel



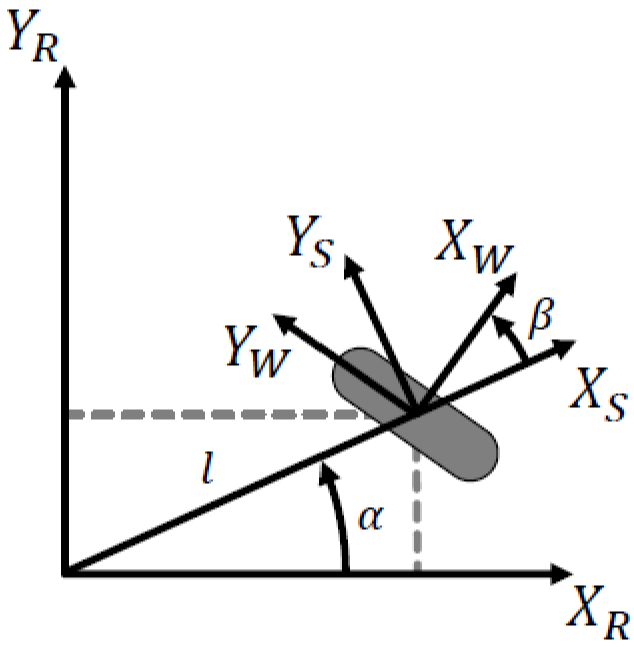
unconstrained rotation about the contact point

$\boxed{\dot{y}_W = r\dot{\varphi}}$ rolling constraint

$\boxed{\dot{x}_W = 0}$ no-sliding constraint

$\boxed{\dot{z}_W = 0}$ planar motion assumption

Standard Wheel (Cont.)



Start with the general equation for a standard wheel

- $\mathbf{v}_{IW} = \mathbf{v}_{IR} + \boldsymbol{\omega}_{IR} \times \mathbf{r}_{RS}$

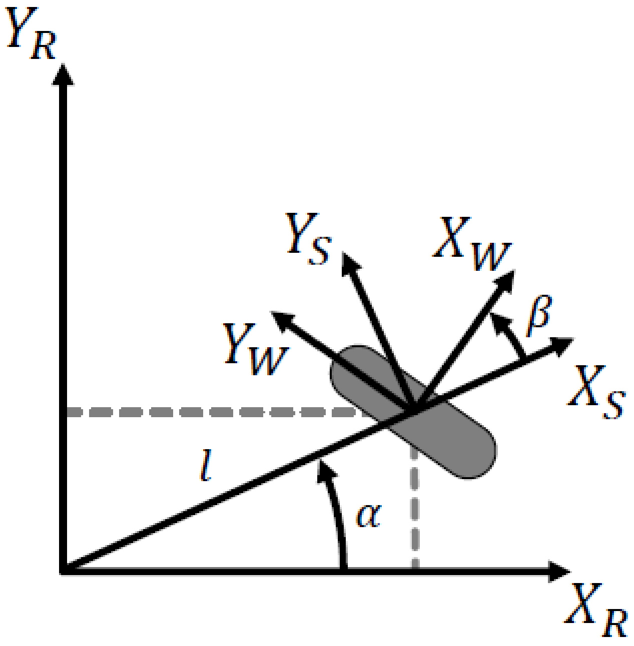
Express this equation in the wheel frame

- ${}_W\mathbf{v}_{IW} = {}_W\mathbf{v}_{IR} + {}_W\boldsymbol{\omega}_{IR} \times {}_W\mathbf{r}_{RS}$

The left hand side is known

- ${}_W\mathbf{v}_{IW} = \begin{bmatrix} 0 \\ \dot{\varphi} r \\ 0 \end{bmatrix}$ - no-sliding constraint
  - rolling constraint
  - planar assumption

Standard Wheel (Cont.)



- $_W\mathbf{v}_{IW} = {}_W\mathbf{v}_{IR} + {}_W\boldsymbol{\omega}_{IR} \times {}_W\mathbf{r}_{RS}$
- $_W\mathbf{v}_{IR} = R_{WS}R_{SR}R_{RI}\begin{bmatrix}\dot{x}\\ \dot{y}\\ 0\end{bmatrix} = R(\alpha+\beta)R(\theta)\begin{bmatrix}\dot{x}\\ \dot{y}\\ 0\end{bmatrix}$
- $_W\boldsymbol{\omega}_{IR} \times {}_W\mathbf{r}_{RS} = \begin{bmatrix}0 & -\dot{\theta} & 0\\ \dot{\theta} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}l\cos\beta\\ -l\sin\beta\\ 0\end{bmatrix} = \begin{bmatrix}l\sin\beta\\ l\cos\beta\\ 0\end{bmatrix}\dot{\theta}$
- Rolling constraint

$$[-\sin\alpha+\beta \quad \cos\alpha+\beta \quad l\cos\beta]R(\theta)\dot{\xi}_I - \dot{\varphi}r = 0$$

$$J_1(\beta_s)R(\theta)\dot{\xi}_I - \dot{\varphi}r = 0$$

- No-sliding constraint

$$[\cos\alpha+\beta \quad \sin\alpha+\beta \quad l\sin\beta]R(\theta)\dot{\xi}_I = 0$$

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0$$

Differential Kinematics

- Given a wheeled robot, each wheel imposes n constraints, **only fixed and steerable standard wheels impose no-sliding constraints**. Suppose a robot has n wheels of radius r_i, the individual wheel constraints can be concatenated in matrix form
- Rolling Constraints

$$J_1(\beta_s)R(\theta)\dot{\xi}_I - J_2\dot{\varphi} = 0$$

$$J_1(\beta_s) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}, J_2 = diag(r_1, \ldots, r_n), \dot{\varphi} = \begin{bmatrix} \dot{\varphi}_1 \\ \vdots \\ \dot{\varphi}_n \end{bmatrix}$$

- No-Sliding Constraints

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0$$

$$C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

Differential Kinematics

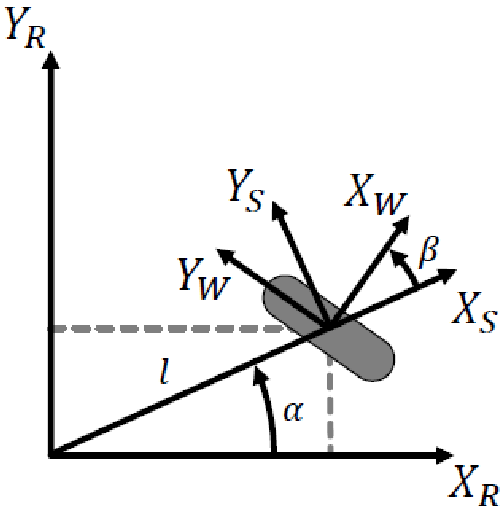
- Stacking the rolling and no-sliding constraints gives an expression for the differential kinematics

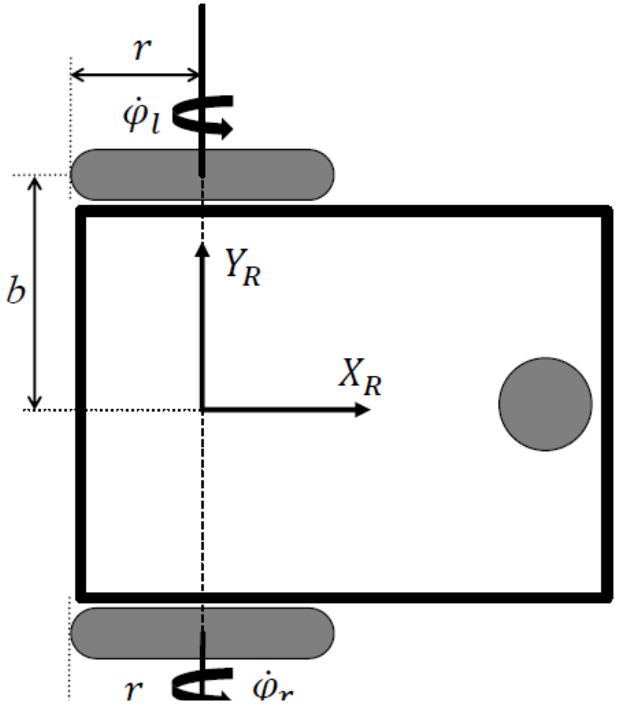
$$\begin{bmatrix} J_1(\beta_s) \\ C_1(\beta_s) \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2 \\ 0 \end{bmatrix} \dot{\varphi}$$

- Solving this equation for $\dot{\xi}_I$ yields the **forward differential kinematics** equation needed for computing wheel odometry
- Solving this equation for $\dot{\varphi}$ yields the **inverse differential kinematics** needed for control

A Differential Drive Robot (Example)

- Two fixed standard wheels
- The robot frame (R) in between the wheels
- Stack the wheel equations for this configuration





For the right wheel

$$\alpha = -\pi/2, \beta = 0, l = b$$

For the left wheel

$$\alpha = -\pi/2, \beta = 0, l = -b$$

A Differential Drive Robot (Example)

For the right wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = b$$

For the left wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = -b$$

Rolling constraint

$$[-\sin\alpha+\beta \quad \cos\alpha+\beta \quad l\cos\beta]\dot{\xi}_R = \dot{\varphi}.$$

$$\begin{bmatrix}1 & 0 & b\\ 1 & 0 & -b\end{bmatrix}\dot{\xi}_R = \begin{bmatrix}r & 0\\ 0 & r\end{bmatrix}\begin{bmatrix}\dot{\varphi}_r\\ \dot{\varphi}_l\end{bmatrix}$$

No-sliding constraint

$$[\cos\alpha+\beta \quad \sin\alpha+\beta \quad l\sin\beta]\dot{\xi}_R = 0$$

$$\begin{bmatrix}0 & -1 & 0\\ 0 & -1 & 0\end{bmatrix}\dot{\xi}_R = \begin{bmatrix}0\\ 0\end{bmatrix}$$

Stacked equations of motion

$$\underbrace{\begin{bmatrix}1 & 0 & b\\ 1 & 0 & -b\\ 0 & -1 & 0\\ 0 & -1 & 0\end{bmatrix}}_{=:A}\dot{\xi}_R = \underbrace{\begin{bmatrix}r & 0\\ 0 & r\\ 0 & 0\\ 0 & 0\end{bmatrix}}_{=:B}\underbrace{\begin{bmatrix}\dot{\varphi}_r\\ \dot{\varphi}_l\end{bmatrix}}_{=:\dot{\varphi}}$$

$$A\dot{\xi}_R = B\dot{\varphi} \qquad \underbrace{A}_{4\times3}\ \dot{\xi}_R = \underbrace{B}_{4\times2}\ \dot{\varphi}$$

$$J_1 = \begin{bmatrix}1 & 0 & b\\ 1 & 0 & -b\end{bmatrix}$$

$$C_1 = \begin{bmatrix}0 & -1 & 0\\ 0 & -1 & 0\end{bmatrix}$$

Differential Drive Forward Kinematics

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ b & -b & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix} (\mathrm{A^T A})^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2b^2 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ b & -b & 0 & 0 \end{bmatrix} \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix}$$

Differential Drive Forward Kinematics

$$(\mathrm{A}^{\mathrm{T}}\mathrm{A})^{-1}A^{T}B = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2b^{2} \end{bmatrix}\begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix}$$

Forward kinematics solution

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix}\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

Forward velocity: $\dot{x} = r\frac{(\dot{\varphi}_r+\dot{\varphi}_l)}{2}$

No-sliding: $\dot{y} = 0$

Angular velocity: $\dot{\theta} = r\frac{(\dot{\varphi}_r-\dot{\varphi}_l)}{2b}$

Differential Drive Robot Inverse Kinematics

Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$B^T B = \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \end{bmatrix} \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix} \quad (B^T B)^{-1} = \begin{bmatrix} 1/r^2 & 0 \\ 0 & 1/r^2 \end{bmatrix}$$

$$B^T A = \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} r & 0 & br \\ r & 0 & -br \end{bmatrix}$$

$$(B^T B)^{-1} B^T A = \begin{bmatrix} 1/r^2 & 0 \\ 0 & 1/r^2 \end{bmatrix} \begin{bmatrix} r & 0 & -br \\ r & 0 & br \end{bmatrix} = \begin{bmatrix} 1/r & 0 & b/r \\ 1/r & 0 & -b/r \end{bmatrix}$$

$$\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix} = \begin{bmatrix} 1/r & 0 & b/r \\ 1/r & 0 & -b/r \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

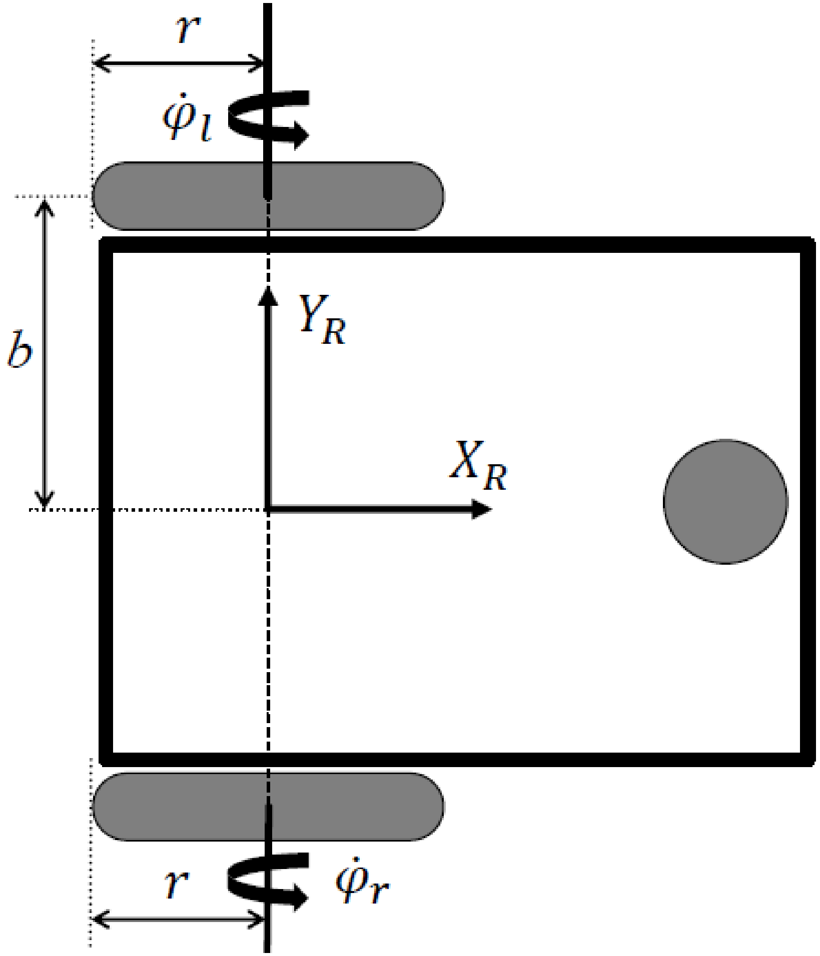
Differential Drive Robot Kinematics (Summary)

Forward differential kinematics

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

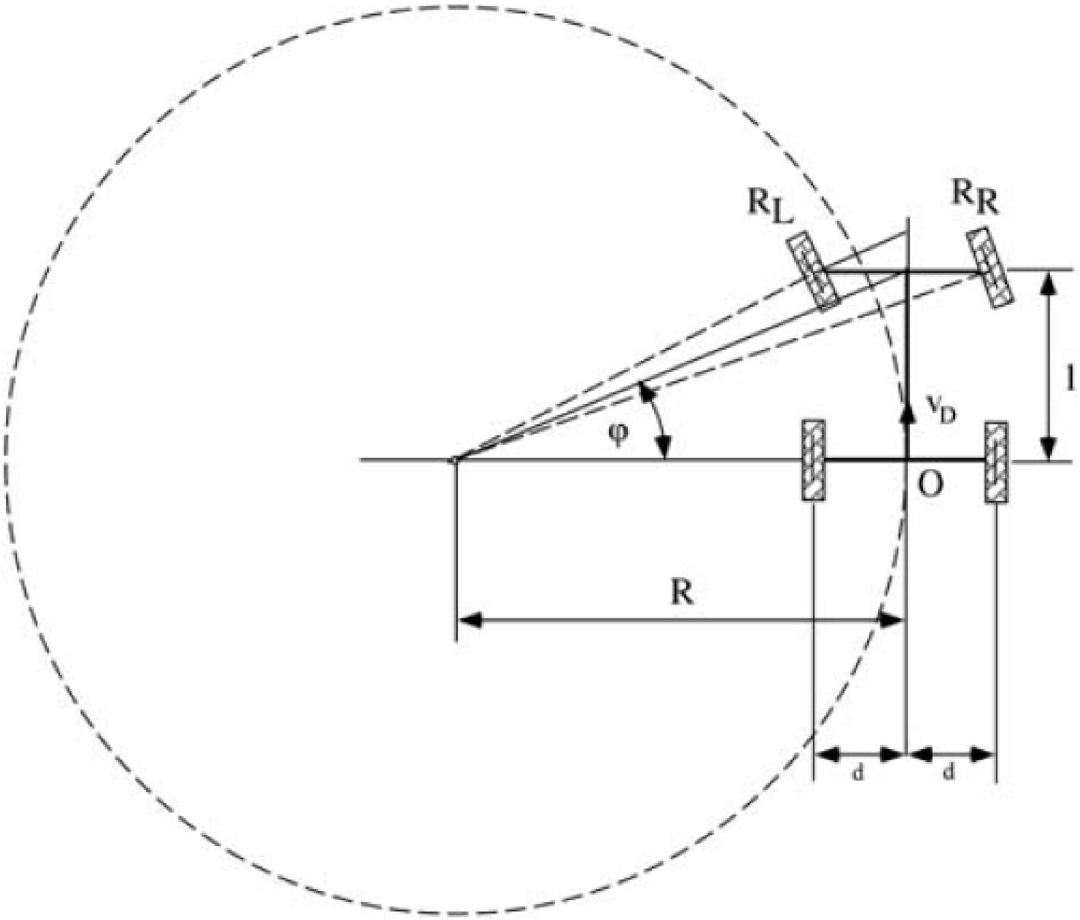
Inverse differential kinematics

$$\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix} = \begin{bmatrix} 1/r & 0 & b/r \\ 1/r & 0 & -b/r \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$



Ackermann Steering

- This specific type of drive system is mostly found in the field of automotive applications.
- It consist of a fixed axle and another one connecting the parallel steered wheels
- If driven wheels are connected to fixed axle, differential system is require , e.g. car, if steered wheels are driven then differential is obsolete.
- It has 3 DOF which are not independent.



Ackermann Steering

- Desired drive speed is denoted by $\boldsymbol{V_D}$ while $\boldsymbol{V_{RR}}, \boldsymbol{V_{LR}}$ are rear right and left wheel speed and $\boldsymbol{V_{RF}}, \boldsymbol{V_{LF}}$ are front right and left wheel speed
- $\boldsymbol{l}$ denotes the length of vehicle, $\boldsymbol{d}$ denotes the distance between wheel and kinematic center and $\boldsymbol{\varphi}$ is steer angle.

$$R = \frac{l}{\tan\varphi}$$

$$v_{LR} = \frac{(R-d)\cdot v_D}{R}$$

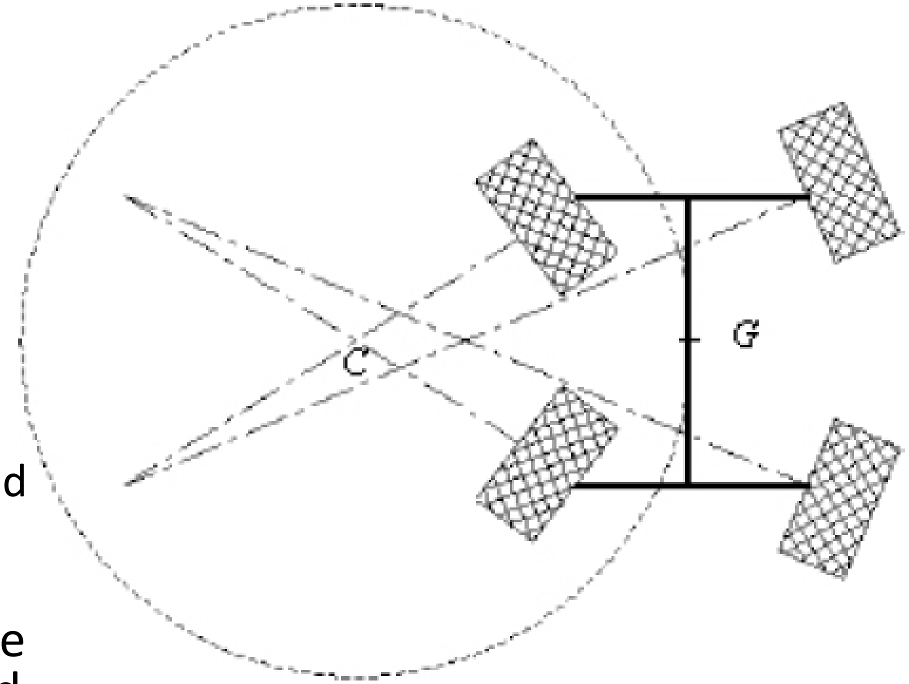
$$v_{RR} = \frac{(R+d)\cdot v_D}{R}$$

$$v_{LF} = \frac{\sqrt{(R-d)^2 + l^2}\cdot|\tan\varphi|}{l}\cdot v_D$$

$$v_{RF} = \frac{\sqrt{(R+d)^2 + l^2}\cdot|\tan\varphi|}{l}\cdot v_D$$

Double Ackermann Steering

- In a double Ackermann steering both axles are steerable
- It is kinematically even more complex an problematic then the Ackermann steering
- When a curve is steered, two rotation points of the robot motion will occur. This yields slip of the single wheels.
- The advantages are
  - Smaller turning radius
  - Sideward motion(both axle are steered in parallel)
- In off-road applications, the errors of this configuration are lower than those of the interaction between vehicle and terrain



Double Ackermann Steering

- Using the vehicle parameters and desired velocity and steering angles the turning radius and wheel velociis are as follows

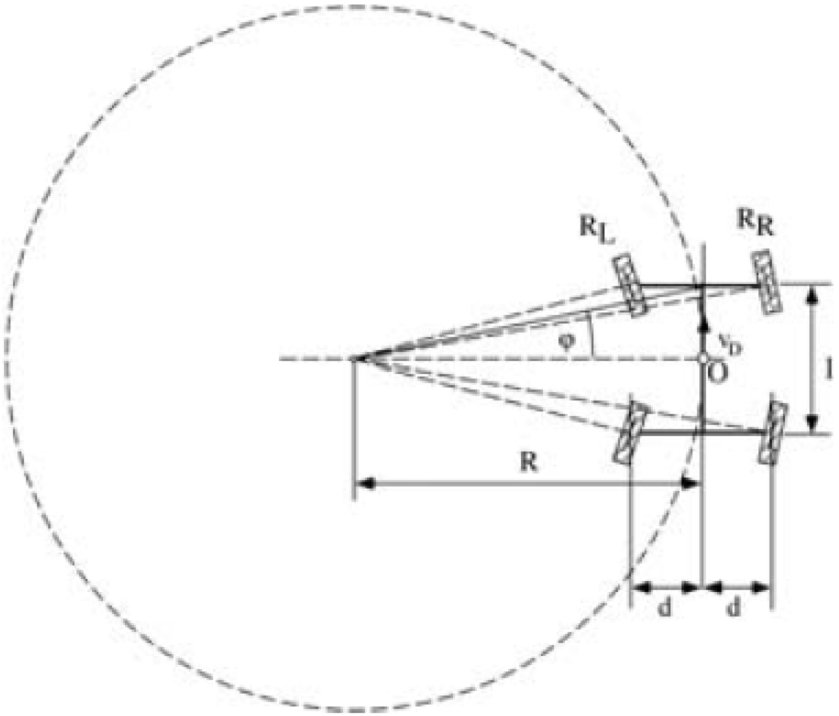
$$R = \frac{l}{\tan\varphi}$$

$$v_{LR} = \frac{\sqrt{(\frac{R}{2}-d)^2+\frac{l}{4}^2}\cdot 2|\tan\varphi|}{l}\cdot v_D$$

$$v_{RR} = \frac{\sqrt{(\frac{R}{2}+d)^2+\frac{l}{4}^2}\cdot 2|\tan\varphi|}{l}\cdot v_D$$

$$v_{LF} = \frac{\sqrt{(\frac{R}{2}-d)^2+\frac{l}{4}^2}\cdot 2|\tan\varphi|}{l}\cdot v_D$$

$$v_{RF} = \frac{\sqrt{(\frac{R}{2}+d)^2+\frac{l}{4}^2}\cdot 2|\tan\varphi|}{l}\cdot v_D$$

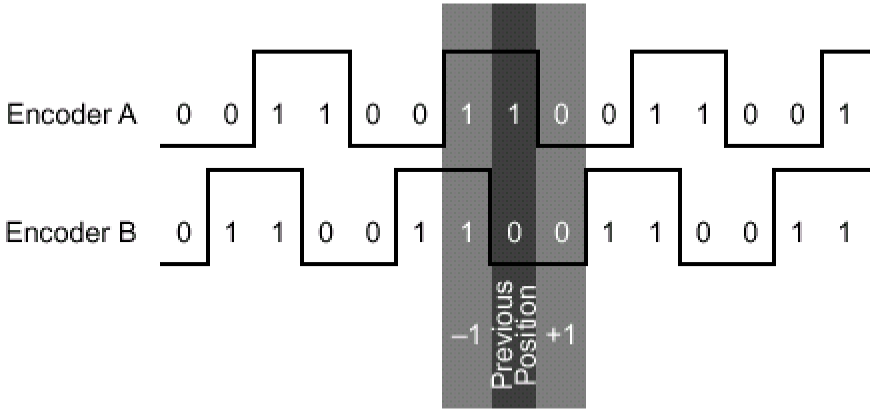


Introduction to mobile robot sensors

- Wheel Encoders
- Range Sensors
  - Ultrasonic
  - Infrared
  - 2D/3D Laser Range Scanner
- Inertial Measurement Unit (IMU)
  - Gyroscope
  - Accelerometer
  - Magnetometer
- Global Positioning System (GPS)
- Vision Sensor
  - Monocular camera
  - Stereo cameras

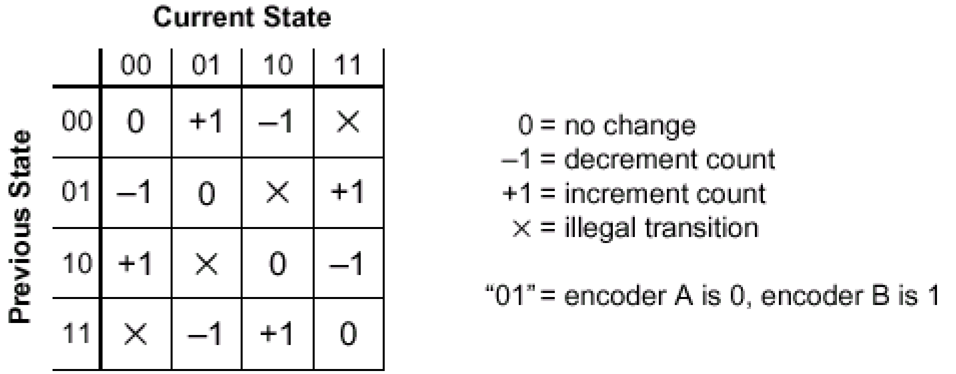
Wheel Encoders

- A pair of encoders is used on a single shaft. The encoders are aligned so that their two data streams are one quarter cycle (90 deg.) out of phase.
- Which direction is shaft moving?
  - Suppose the encoders were previously at the position highlighted by the dark band; i.e., Encoder A as 1 and Encoder B as 0. The next time the encoders are checked:
  - If they moved to the position AB=00, the position count is incremented
  - If they moved to the position AB=11, the position count is decremented



Wheel Encoders (Cont.)

- State transition table
  - Previous state and current state are the same, then there has been no change in position
  - Any single-bit change corresponds to incrementing/decrementing the count
  - If there is a double-bit change, this corresponds to the encoders being misaligned, or having moved too fast in between successive checks—an illegal transition



Current State

Previous State	00	01	10	11
00	0	+1	–1	×
01	–1	0	×	+1
10	+1	×	0	–1
11	×	–1	+1	0

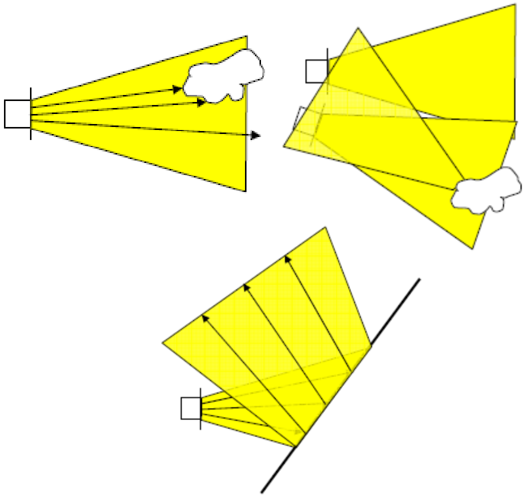
0 = no change
–1 = decrement count
+1 = increment count
× = illegal transition

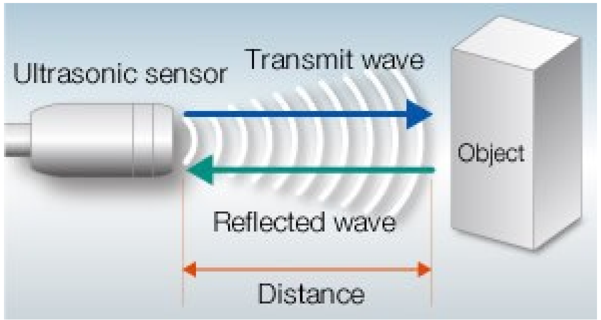
"01" = encoder A is 0, encoder B is 1

Ultrasonic

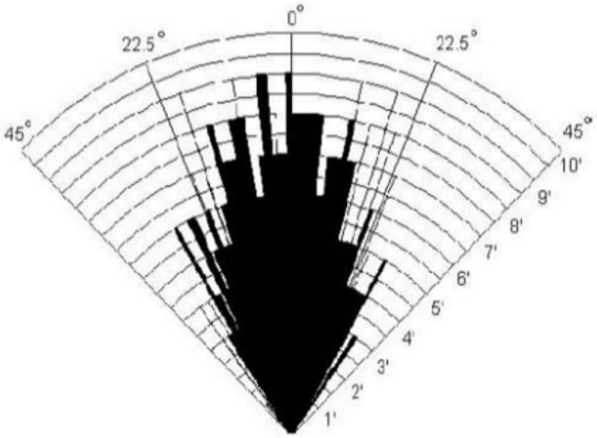
- Active time of flight sensor, emit an ultrasound signal and wait until it receive the echo

- Opening angle, crosstalk, specular reflection





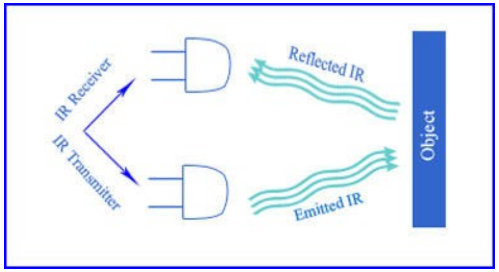
$$d = v \times t / 2$$

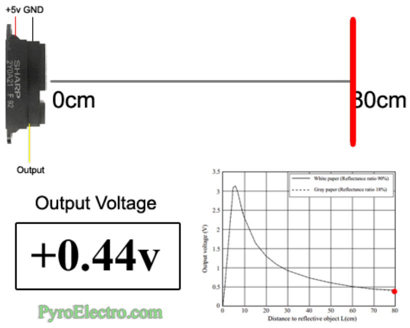


Practical test of performance,
Best in 30 degree angle

Infrared

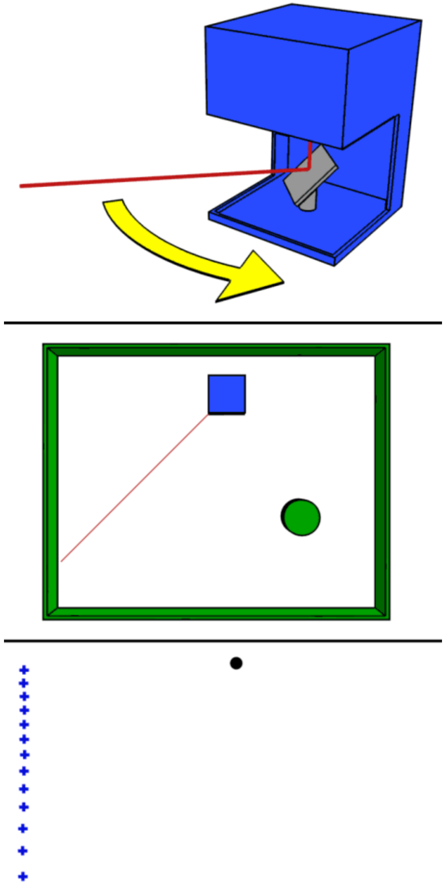
- An IR proximity sensor works by applying a voltage to a pair of IR light emitting diodes (LED’s) which in turn, emit infrared light. This light propagates through the air and once it hits an object it is reflected back towards the sensor. If the object is close, the reflected light will be stronger than if the object is further away.
- It has problem associated with the color of the surface



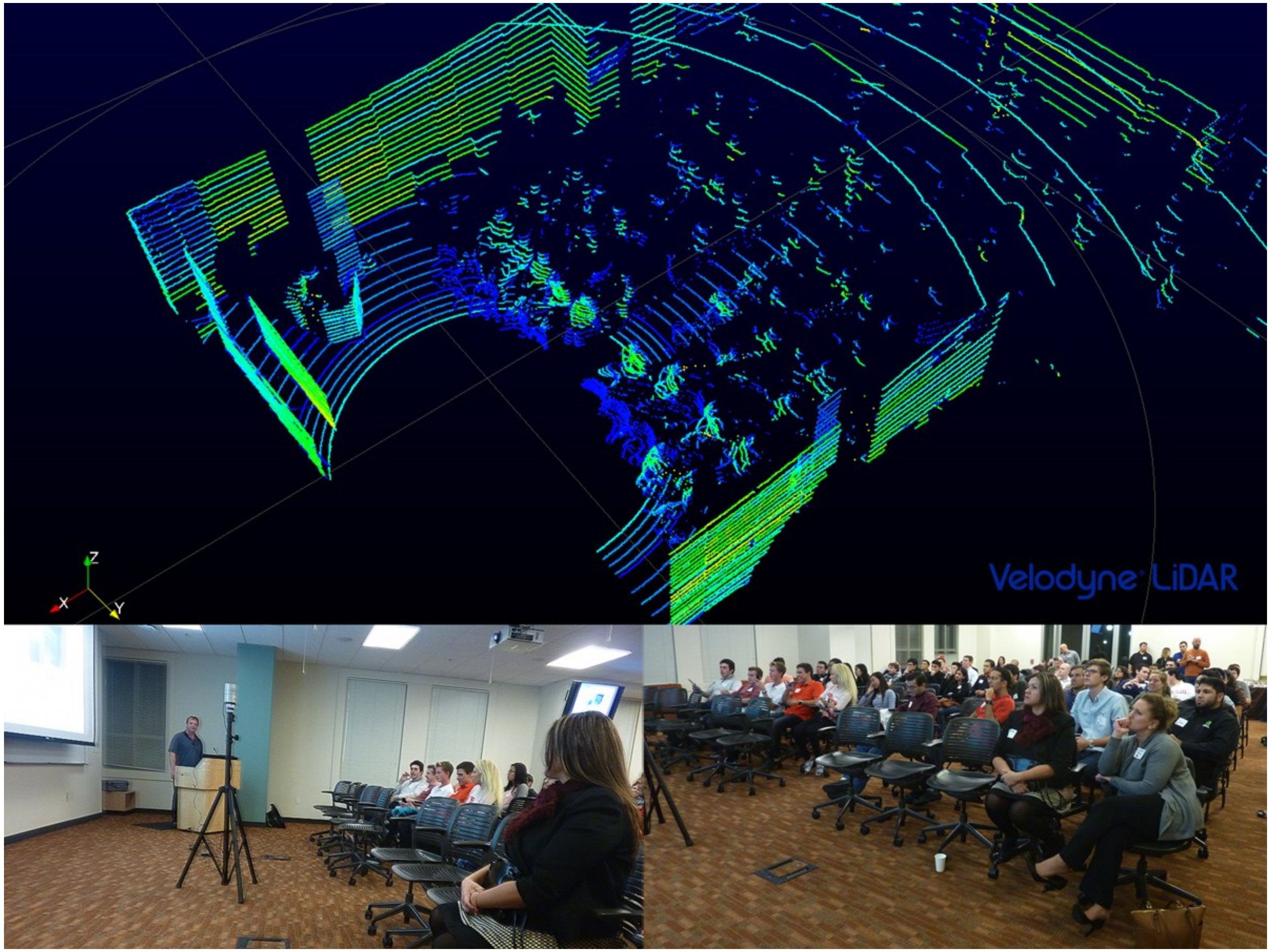


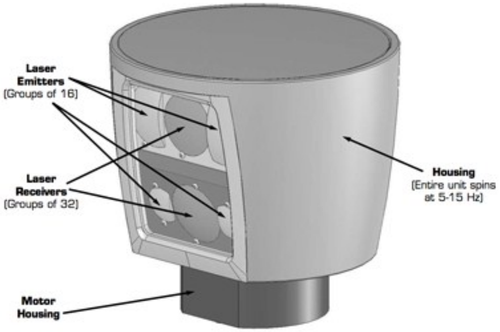
2D Laser Range Scanner

- High Precision
- Wide field of view



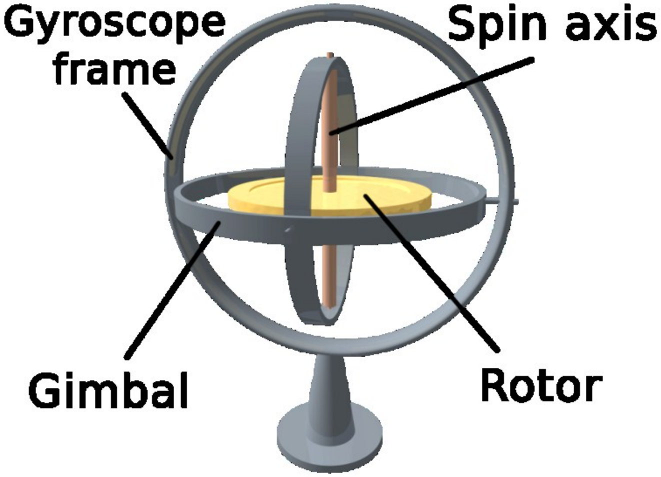
3D Laser Range Scanner

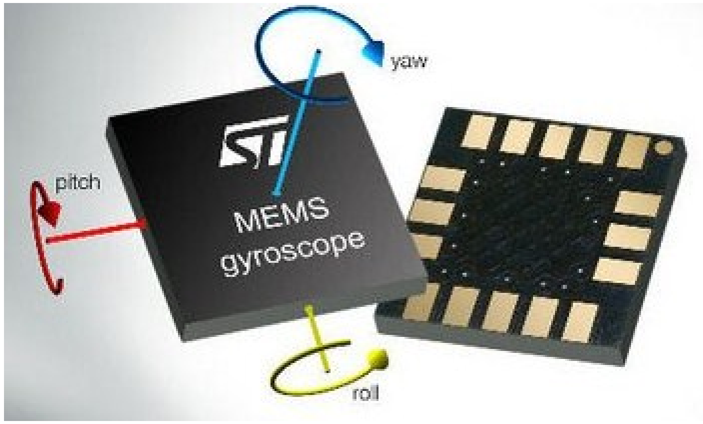




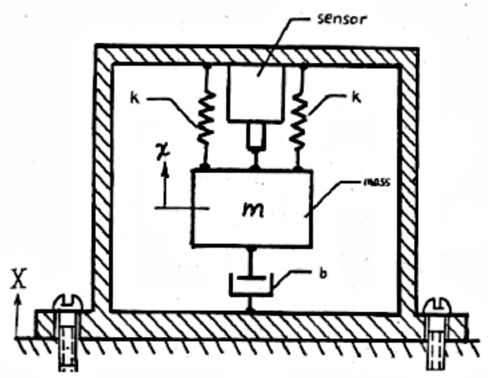
Gyroscope

- A gyroscope is a device for measuring or maintaining orientation, based on the principles of conservation of angular momentum
- Measures orientation (standard gyro) or angular velocity (rate gyro, needs integration for angle)

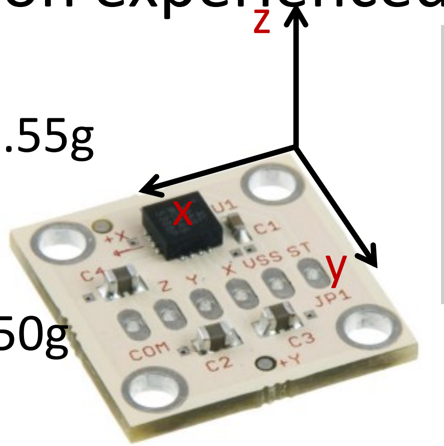


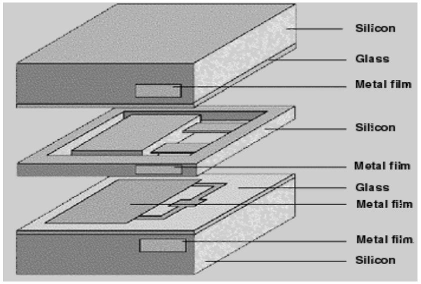


Accelerometer



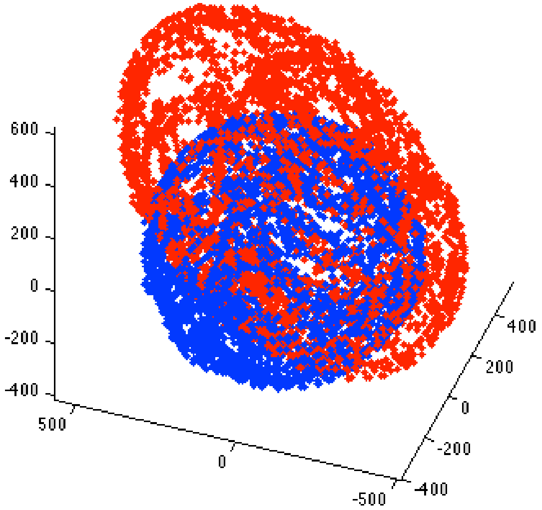
- Measures all external forces acting upon them (including gravity)
- To obtain inertial acceleration (due to motion alone), gravity must be subtracted
- Accelerometers behave as a damped mass on a spring. Acceleration causes displacement of this "spring" proportional to the acceleration experienced.
- This room = your weight = 1g
- Bugatti Veyron, 0 to 100Km/h in 2.4s= 1.55g
- Space Shuttle reentry & launch = 3g
- Max experienced by a human* = 46.2g
  Death or extensive & severe injuries= +50g

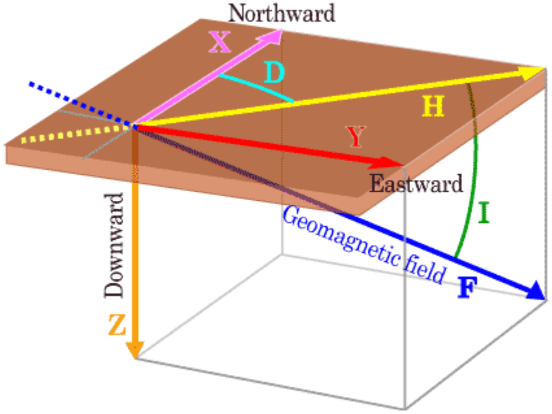


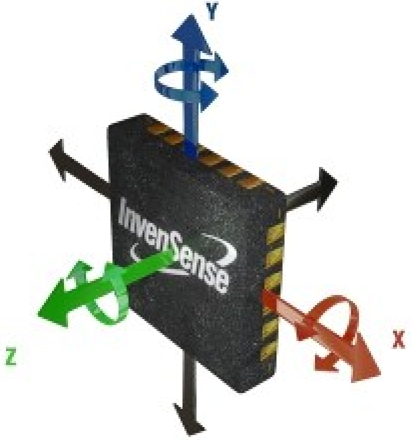


Magnetometer

- Compass invented by the Chinese in the 4th century, Carl Gauss invents the "magnetometer" in 1833
- Earth magnetic field, Hard and Soft Iron effects

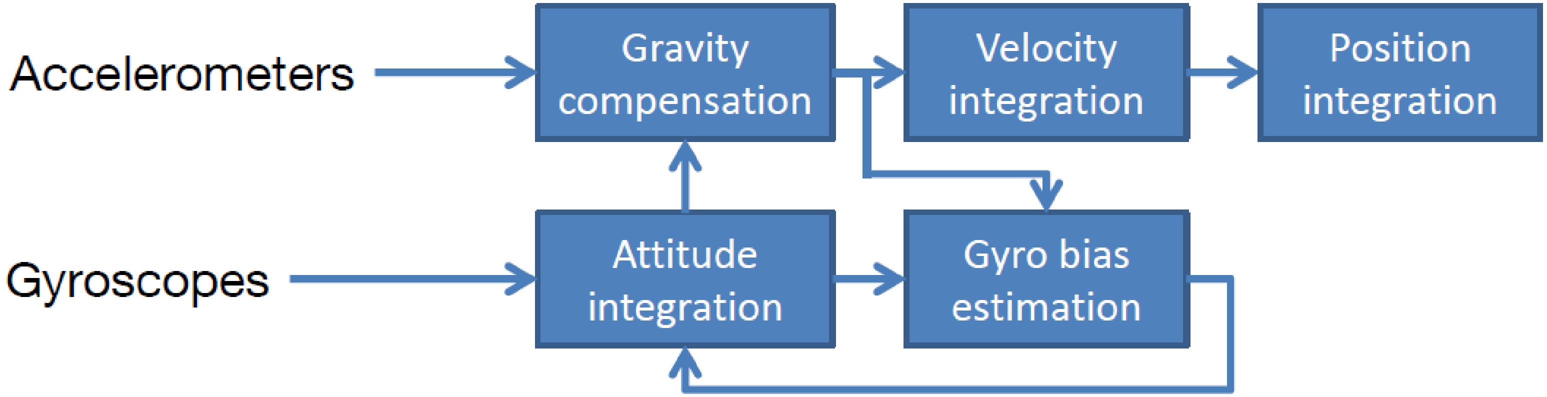






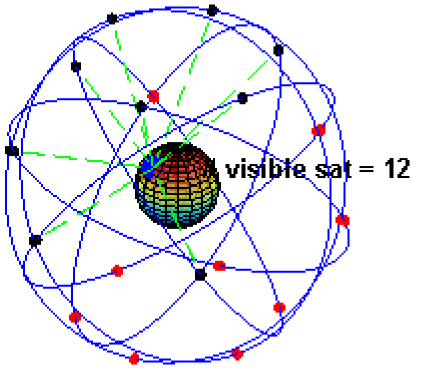
IMU

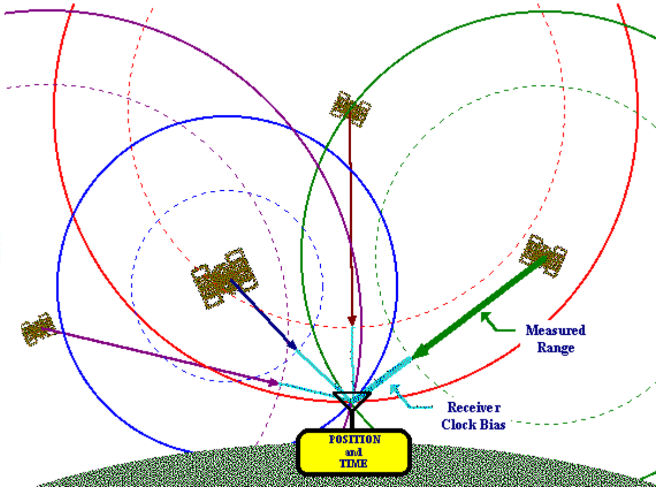
- 3-axes MEMS gyroscope
  - Provides angular velocity
  - Integrate for angular position
- 3-axes MEMS accelerometer
  - Provides accelerations (including gravity)

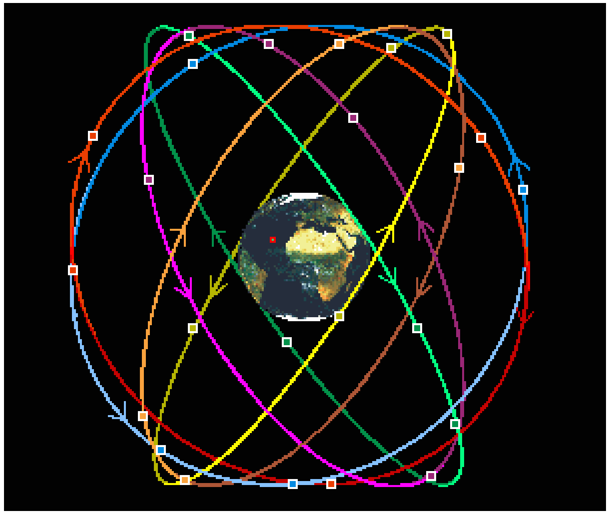


Global Positioning System

- 24+ satellites, 12 hour orbit, 20.190 km height
- 6 orbital planes, 4+ satellites per orbit, 60deg distance
- Every satellite transmits its position and time
- Requires measurements of 4 different satellites
- Low accuracy (3-15m) but absolute

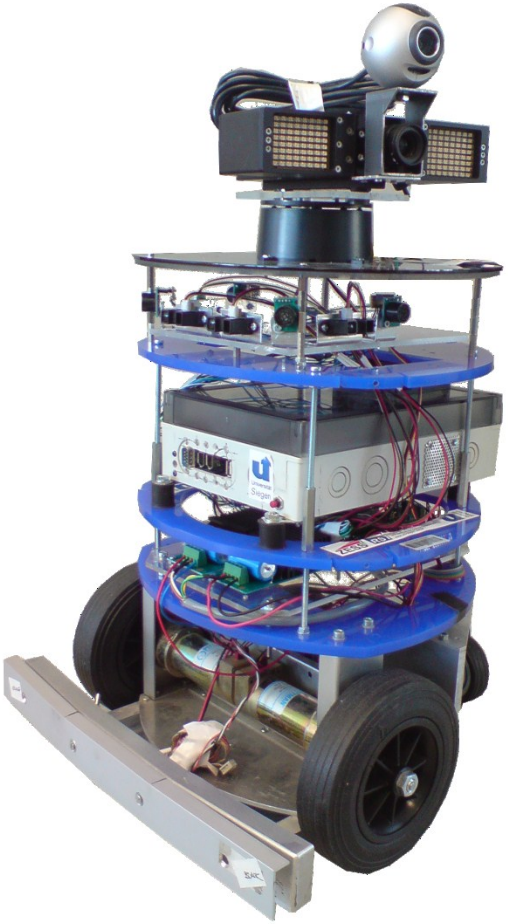






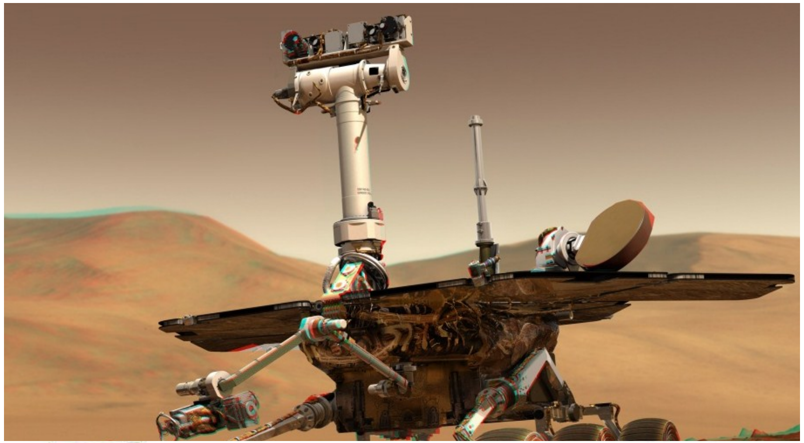
Monocular Camera

- Vision is most powerful sense
- CCD and CMOS
- Automatic extraction of meaningful information (features)
- Applications
  - 3D reconstruction and modeling
  - Motion capture
  - Teleportation
  - Robot navigation
- Problems
  - Blurr (Aperature)
  - Lense Distortation
  - Projective geometry (length, angles)



Stereo Camera

- It is the process of obtaining depth information from a pair of images coming from two cameras that look at the scene from different but known position
- Correspondence search prblem

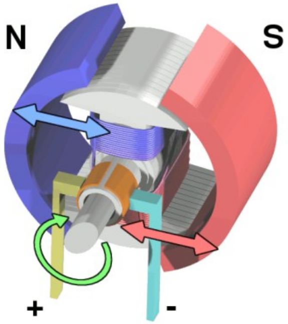


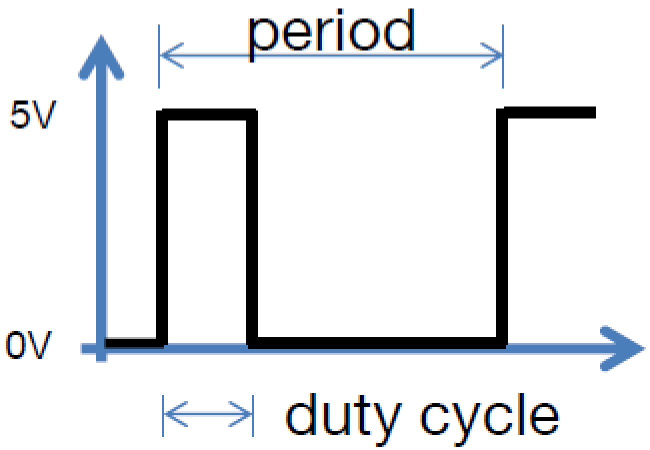
Introduction to mobile robot actuators

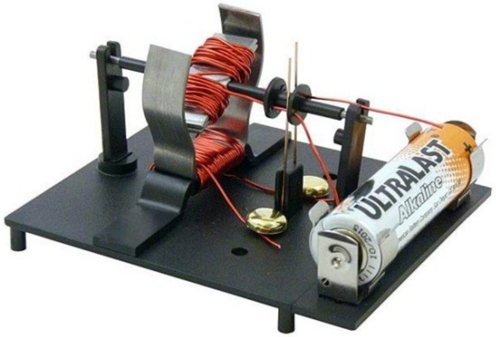
- DC Motors
  - Brush motor
  - Brushless motor
- Motion control
  - Open-loop control (trajectory following)
  - Feedback control
  - PID based velocity/position control

DC Brush Motor

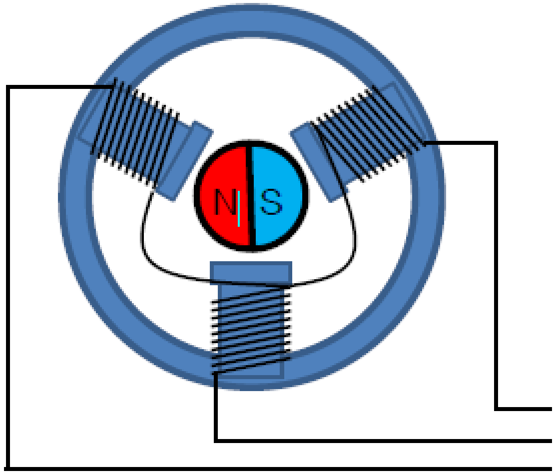
- More power = faster rotation
- Power is modulated using PWM



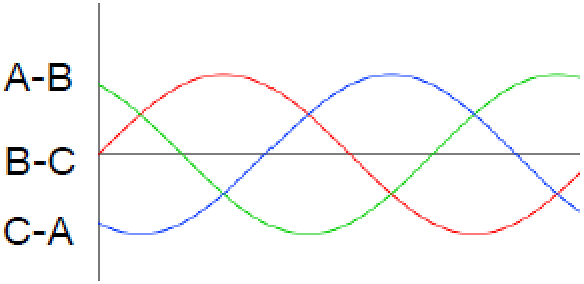


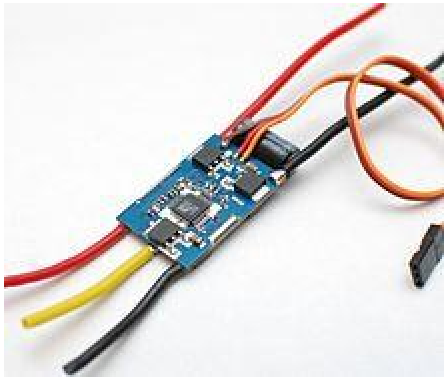


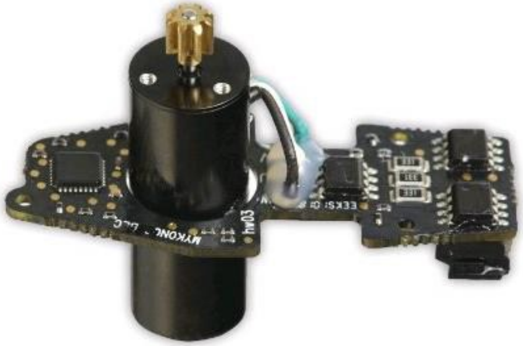
DC Brushless Motor



- Electromagnets are stationary
- Permanent magnets on the axis
- No brushes (less maintenance, higher efficiency)
- Measure motor position/speed using back-EMF





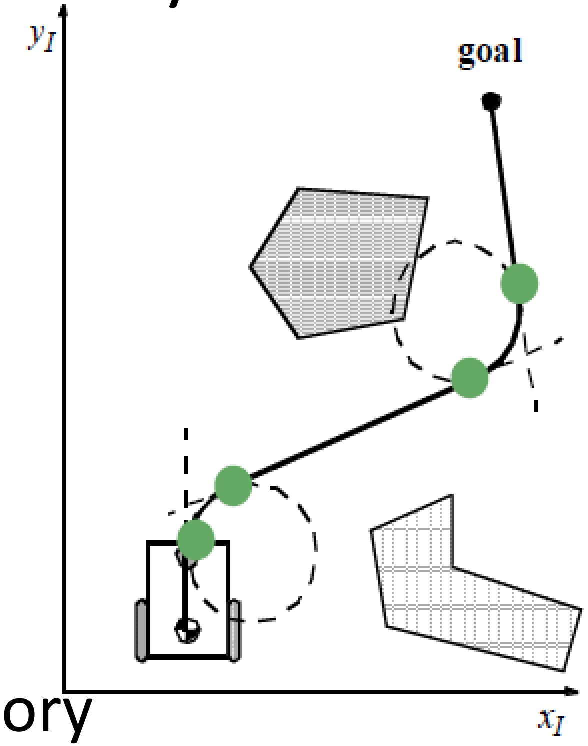


Motion Control

- The objective of a kinematic controller is to follow a trajectory described by its position and/or velocity profiles as function of time.
- Motion control is not straight forward because mobile robots are typically non-holonomic systems.
- Most controllers are not considering the dynamics of the system

Open-loop Control

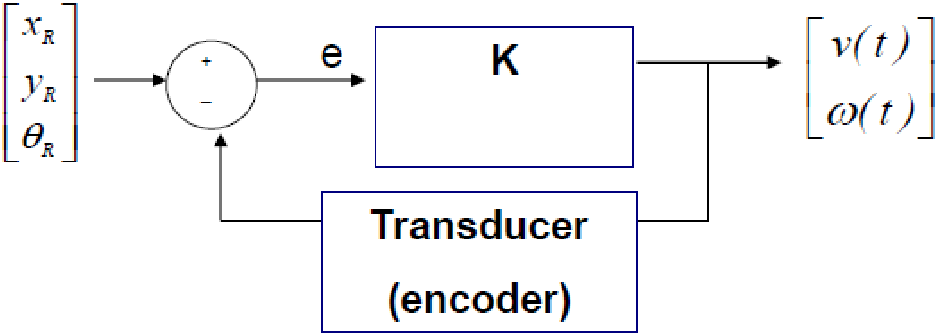
- Trajectory divided in segments of clearly defined shape
  - Lines and arcs
- control problem
  - pre-compute a smooth trajectory based on line and arcs
- Disadvantages
  - limitations and constraints of the robots velocities and accelerations
  - does not adapt or correct the trajectory if dynamical changes of the environment occur



Feedback Control

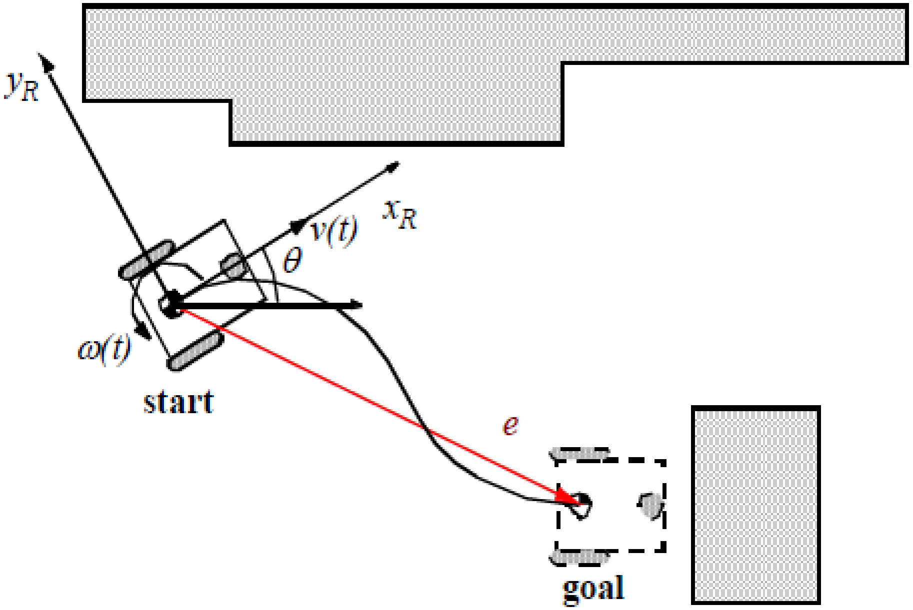
- Find a control matrix K,

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$$



such that the control of $v(t)$ and $\omega(t)$ drives the error e to zero

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \cdot {}^{R}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$



Position Control

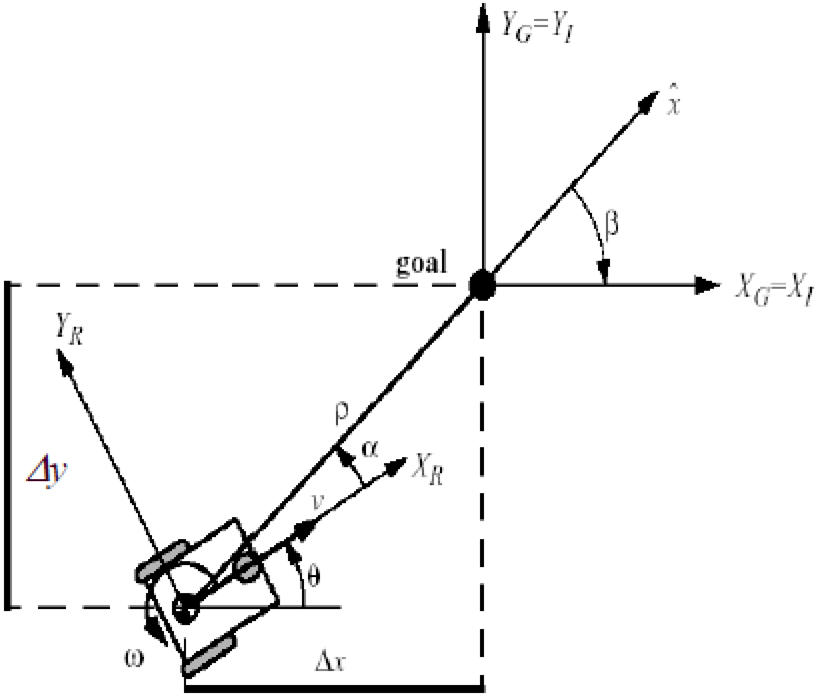
- Assume that the goal of the robot is the origin of the global inertial frame. The *kinematics* for the differential drive mobile robot with respect to the global reference frame are:

$$^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$



Position Control

Robot is facing the goal point

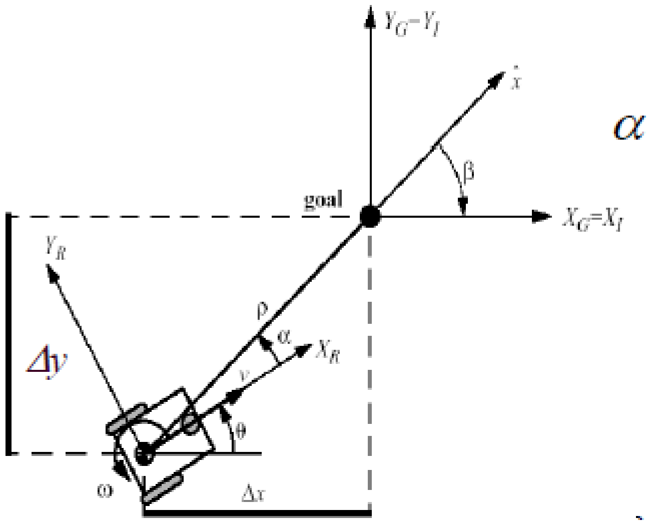
$$\begin{bmatrix}\dot{\rho}\\ \dot{\alpha}\\ \dot{\beta}\end{bmatrix} = \begin{bmatrix}\cos\alpha & 0\\ -\frac{\sin\alpha}{\rho} & 1\\ \frac{\sin\alpha}{\rho} & 0\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}$$

$$\alpha \in (-\pi/2, \pi/2]$$

Robot's back is to the goal point

$$\begin{bmatrix}\dot{\rho}\\ \dot{\alpha}\\ \dot{\beta}\end{bmatrix} = \begin{bmatrix}-\cos\alpha & 0\\ \frac{\sin\alpha}{\rho} & -1\\ -\frac{\sin\alpha}{\rho} & 0\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}$$

$$\alpha \in (\pi/2, \pi] \cup (-\pi, -\pi/2]$$



Position Control (Cont.)

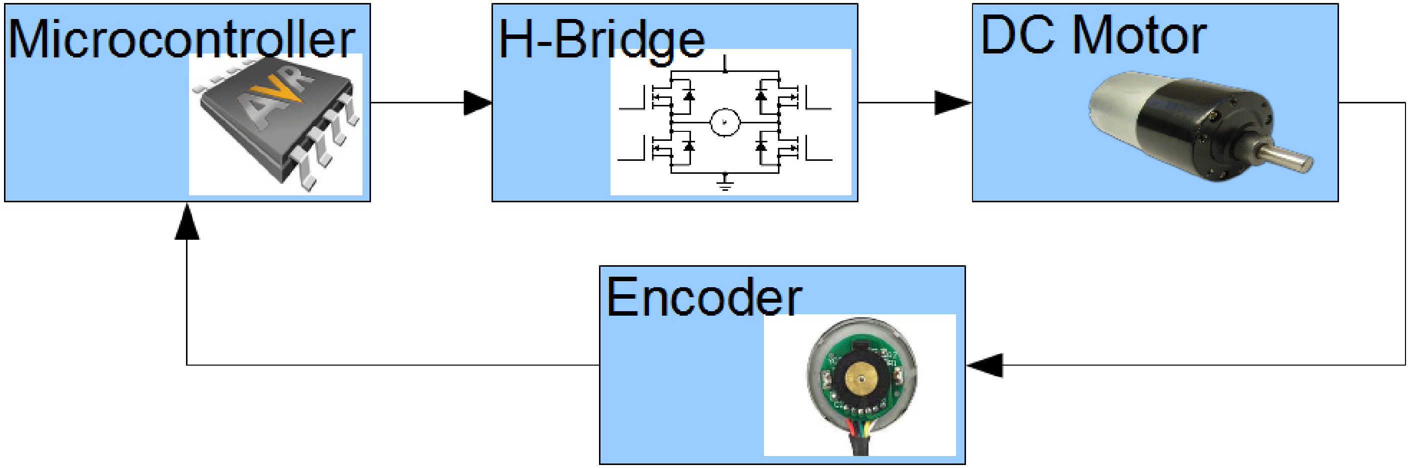
The controls signals v and ω must be designed to drive the robot from $(\rho_o, \alpha_o, \beta_o)$ to the goal position

Consider the control law, $v = k_\rho \rho$ and $\omega = k_\alpha \alpha + k_\beta \beta$

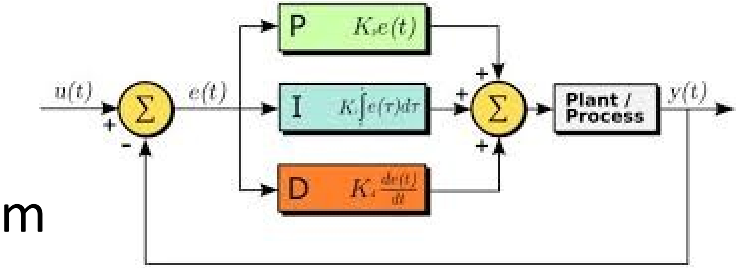
The closed loop system description becomes,

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

PID Velocity Control



- Move the robot at a desired speed?
- Calculate the individual wheel velocities
- Wheel encoders are used for feedback
- Microcontroller runs the control algorithm (PID) and generates required PWM
- H-Bridge is used as power amplifier



Summary

- Wheel Kinematics and Robot Pose calculation
  - Differential wheel drive
  - Ackermann wheel drive
- Introduction to Mobile Robot Sensors
  - Wheel Encoders
  - Inertial Measurement Unit (IMU) and GPS
  - Range sensors (Ultrasonic, 2D/3D Laser Scanner)
  - Vision sensor (Monocular, Stereo Cameras)
- Introduction to Mobile Robot Actuators
  - DC Brush/Brushless motors
  - Position control
  - PID based velocity controller

Questions